CENTRE EUROPÉEN DE RECHERCHE ET DE FORMATION AVANCÉE EN CALCUL SCIENTIFIQUE

Solving linear systems on IBM chips

Presented by

Adrien Suau
adrien.suau@cerfacs.fr
CERFACS, LIRMM

Thesis supervisors

Aida Todri-Sanial
aida.todri@lirmm.fr
LIRMM, CNRS

Gabriel Staffelbach
gabriel.staffelbach@cerfacs.fr
CERFACS

Eric Bourreau
eric.bourreau@lirmm.fr
LIRMM

Marko Rančić
marko.rancic@total.com
TOTAL

Plan of the presentation

1. Use cases at CERFACS and problems of interest
2. The HHL algorithm and the Hamiltonian Simulation problem
3. Variational Quantum Linear Solver (VQLS)
4. Application on IBM chips

Problems targeted

- Computational fluid dynamics:
  - Navier-Stokes equations
- Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u$$

- Helmholtz equation:

$$\Delta f = -k^2 f$$

Classes of problems

- Partial differential equations

$$\frac{\partial^2 u}{\partial t^2} = c^2 \Delta u$$

- Eigenvalue problems

$$\Delta f = -k^2 f$$

- Sparse linear system solving (from PDE discretisation)

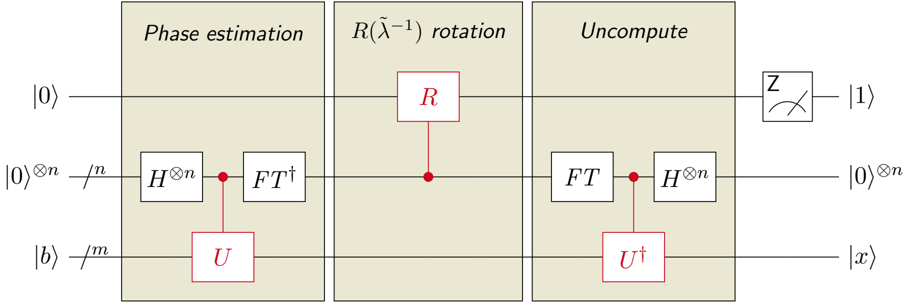
What can be improved?

- Run-time
- Precision of final solution
- Size of solvable problem

First target: HHL algorithm

- Quantum algorithm to solve sparse linear systems
- Scales as $\log(N)$
- Main goal: resources estimation of solving a PDE

Aram W. Harrow, Avinatan Hassidim, and Seth Lloyd. "Quantum Algorithm for Linear Systems of Equations". In: *Phys. Rev. Lett.* 103 (15 Oct. 2009), p. 150502. DOI: 10.1103/PhysRevLett.103.150502


Phase estimation
$R(\tilde{\lambda}^{-1})$ rotation
Uncompute
$|0\rangle$
$|0\rangle^{\otimes n}$
$|b\rangle$
n
m
$H^{\otimes n}$
$FT^\dagger$
U
R
FT
$U^\dagger$
$H^{\otimes n}$
Z
$|1\rangle$
$|0\rangle^{\otimes n}$
$|x\rangle$

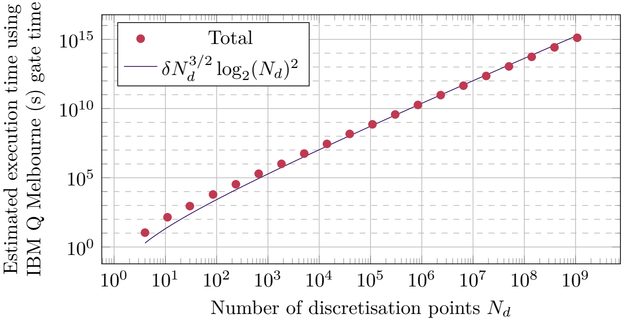
Product formula	Quantum walk	Taylor series
Linear combinations of quantum walk	Trucated Dyson series	Lieb-Robinson bounds
Qubitization	Quantum Signal Processing	Uniform Spectral Amplification

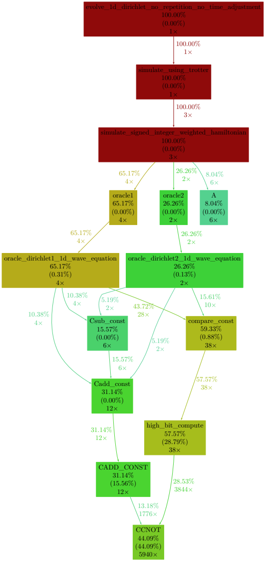
HHL not needed anymore

Pedro C. S. Costa, Stephen Jordan, and Aaron Ostrander. “Quantum algorithm for simulating the wave equation”. In: *Phys. Rev. A* 99 (1 Jan. 2019), p. 012323. DOI: 10.1103/PhysRevA.99.012323

Questions:

1. Does it work?
2. Is it in agreement with theoretical complexity?
3. Is it practically interesting?


Estimated execution time using IBM Q Melbourne (s) gate time
Number of discretisation points N_d
Total
$\delta N_d^{3/2} \log_2(N_d)^2$
10^0
10^5
10^{10}
10^{15}
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9


evolve_1d_dirichlet_no_repetition_no_time_adjustment
simulate_using_trotter
simulate_signed_integer_weighted_hamiltonian
oracle1
oracle2
A
oracle_dirichlet1_1d_wave_equation
oracle_dirichlet2_1d_wave_equation
Csub_const
compare_const
Cadd_const
high_bit_compute
CADD_CONST
CCNOT

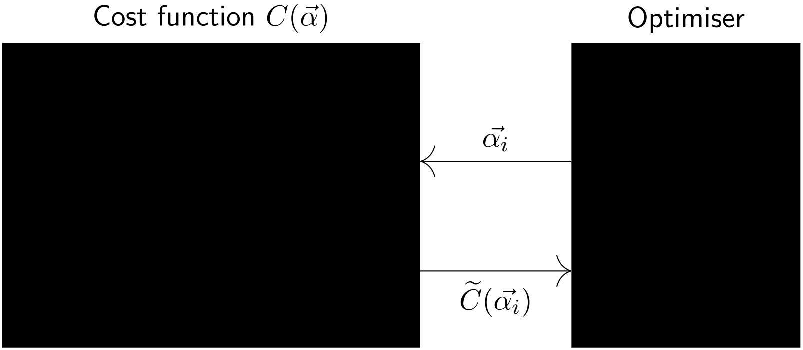
Answers from the implementation analysis:

1. Does it work? Yes
2. Is it in agreement with theoretical complexity? Yes
3. Is it practically interesting? Not yet, a lot of optimisation still required for it to be interesting

Adrien Suau, Gabriel Staffelbach, and Henri Calandra. "Practical Quantum Computing: Solving the Wave Equation Using a Quantum Approach". In: *ACM Transactions on Quantum Computing* 2.1 (Feb. 2021). ISSN: 2643-6809. DOI: 10.1145/3430030

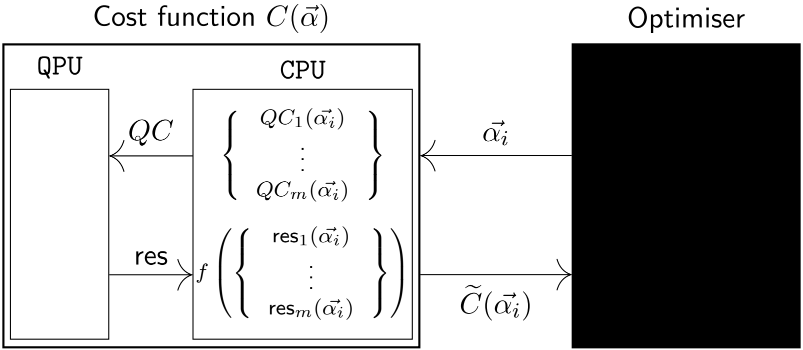
Carlos Bravo-Prieto et al. *Variational Quantum Linear Solver*. 2020. arXiv: 1909.05820 [quant-ph]

- Introduce a **quantum variational algorithm** to solve linear systems of equation.


Cost function $C(\vec{\alpha})$
Optimiser
$\vec{\alpha_i}$
$\widetilde{C}(\vec{\alpha_i})$

Quantum Variational algorithms (QVA)

Bringing quantum to classical optimisation



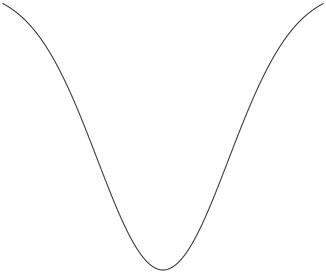
Why using Quantum Variational algorithms?

Desirable properties

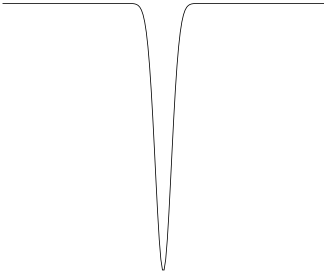
- More resilient to noise
- Only use short and small quantum circuits
- Usable on current Noisy Intermediate Scale Quantum (NISQ) chips

Drawbacks

- No rigorous bounds on run-time or precision
- Barren Plateaus



(a) Cost function – no Barren plateau



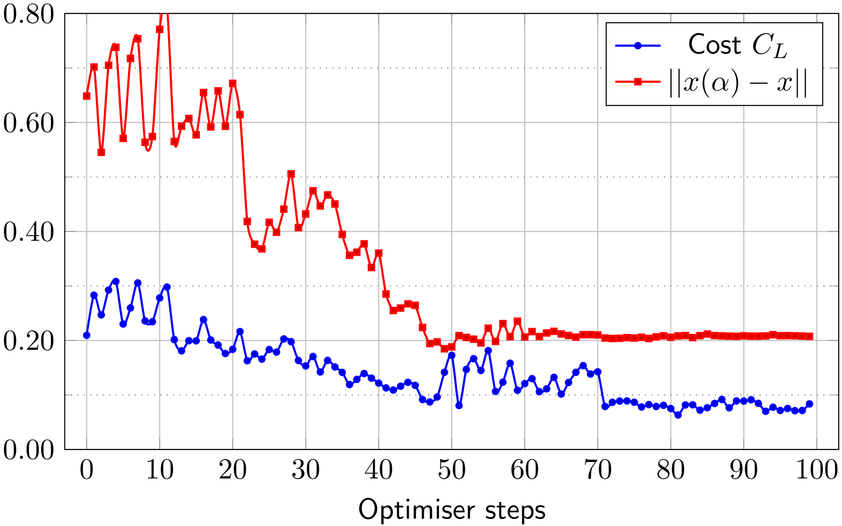
(b) Cost function – Barren plateau

Important choices before running on quantum chips:

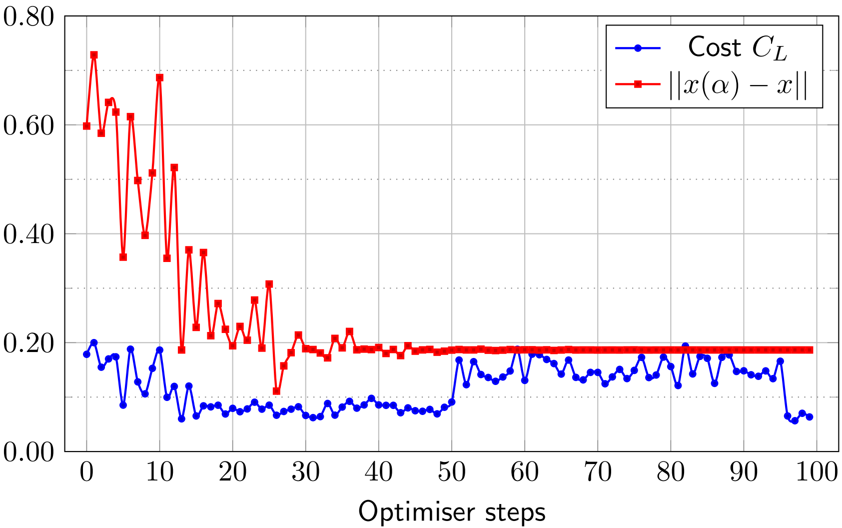
1. Which optimiser?
   - Gradient-free?
   - Gradient-based?
2. Parametrised circuit for state preparation

Sources of noise:

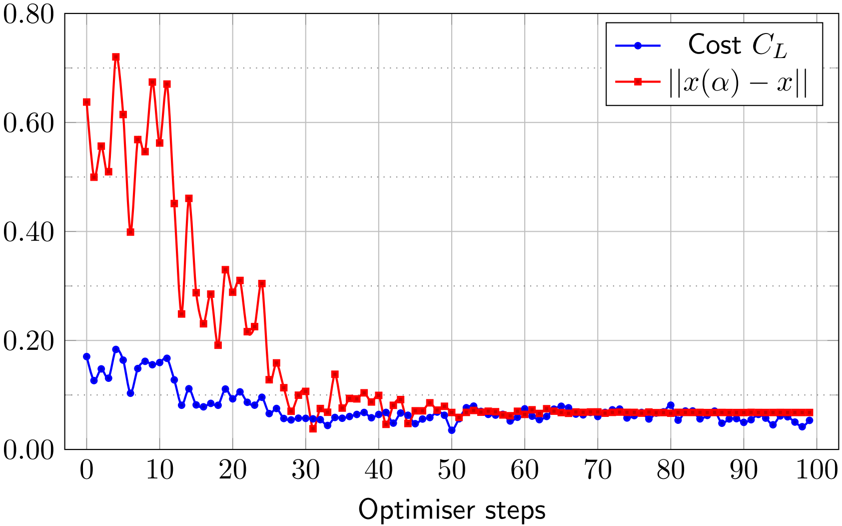
1. Statistical sampling $\approx \sqrt{8192}^{-1} \approx 10^{-2}$
2. Quantum noise:
   - Measurement errors $\approx 2 \times 10^{-2}$
   - Gate errors
     - 1-qubit gates $\approx 10^{-4}$
     - 2-qubit gates $\approx 10^{-2}$
   - Initial state preparation
   - Decoherence

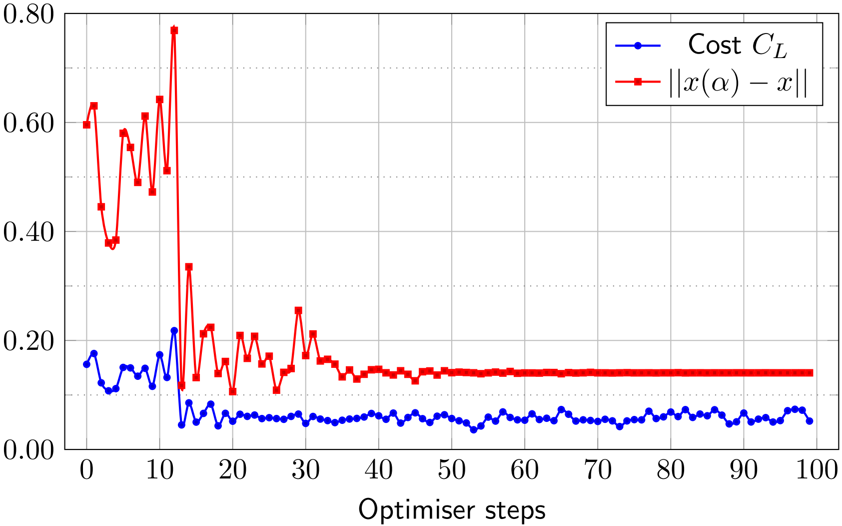


0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60
70
80
90
100
Optimiser steps
Cost C_L
$||x(\alpha) - x||$



0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60
70
80
90
100
Cost C_L
$||x(\alpha) - x||$
Optimiser steps


0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60
70
80
90
100
Optimiser steps
Cost C_L
$||x(\alpha) - x||$



0.80
0.60
0.40
0.20
0.00
0
10
20
30
40
50
60
70
80
90
100
Optimiser steps
Cost C_L
$||x(\alpha) - x||$

Sources of errors:

- Qiskit compiler with optimisation at level 3
- $R_y R_z$ ansatz with linear entanglement

Solutions

- Using optimisation level 2 produce better circuits
- Using R_y ansatz
- Hardware-aware ansatz

Solving linear systems on IBM chips

March 4, 2021

Presented by
Adrien Suau
adrien.suau@cerfacs.fr
CERFACS, LIRMM

https://adrien.suau.me

Any questions?